Statistics

Summer 2023

Lecture 9

Feb 19-8:47 AM

Class QZ 9

Answers in reduced fraction

4 Females, 6 Males, Select 3 people.

1) P(at least 1 Female) = 1 − P(All Males) (No Females)

$= 1 - \frac{6}{10} \cdot \frac{5}{9} \cdot \frac{4}{8} = \boxed{\frac{5}{6}}$ ✓

2) P(at least 1 Male) = 1 − P(All Females) (No Males)

$= 1 - \frac{4}{10} \cdot \frac{3}{9} \cdot \frac{2}{8} = \boxed{\frac{29}{30}}$ ✓

$P(1F \,\&\, 2M) = \frac{{}_4C_1 \cdot {}_6C_2}{{}_{10}C_3} = \frac{60}{120} = \frac{1}{2}$

$P(2F \,\&\, 1M) = \frac{{}_4C_2 \cdot {}_6C_1}{{}_{10}C_3} = \frac{36}{120} = \frac{3}{10}$

Jun 26-11:27 AM

Consider the chart below

x	$P(x)$
1	.05
2	.1
3	.15
4	.2
5	.25
6	.25

1) Find $P(X=6)$

$= 1 - [.05 + .1 + .15 + .2 + .25] = \boxed{.25}$

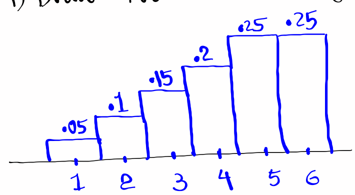
2) Find $P(x \geq 2)$

$= 1 - P(x=1) = 1 - .05 = \boxed{.95}$

3) Find $P(x=2 \text{ or } x=4)$

$= .1 + .2 = \boxed{.3}$

4) Draw Prob. dist. histogram



$x \to L1$, $P(x) \to L2$

use 1-Var Stats with L1 & L2

$\mu = \bar{x} = \boxed{4.25}$

$\sigma = \sigma_x = \boxed{1.479}$

$n = 1$ Total Prob.

Find σ^2 in reduced fraction

VARS, 5: Statistics, 4: σ_x, x^2, Math, 1: ▶Frac, Enter

$\sigma^2 = \frac{35}{16}$

Jun 27-7:39 AM

Round μ & σ to whole numbers

$\mu = 4$ $\sigma = 1$

68% Range → $\mu \pm \sigma$ ⇒ 3 to 5

95% Range → $\mu \pm 2\sigma$ ⇒ 2 to 6

usual Range

99.7% Range → $\mu \pm 3\sigma$ ⇒ 1 to 7

Jun 27-7:49 AM

Expected Value (μ)

25 students, each paid \$10 to buy a ticket.

One ticket drawn, winner gets a calc. worth \$100

what is expected value per ticket sold?

Net gain	P(Net gain)	
10 − 100	1/25	Winning TKT
10 − 0	24/25	Losing TKTS

Net gain → L1

P(Net gain) → L2

Use 1-Var-Stats with L1 & L2

Expected Value per Ticket

$\mu = \bar{x} = 6$

The house (fundraisers) make \$6/TKT.

Jun 27-7:52 AM

You buy insurance for your luggage for \$100.

Any damages, Airline pays you \$1000.

Prob. of any damages is .5% → .005

Find expected value per policy sold by the airline.

Net gain	P(Net gain)	
100 − 1000	.005	Any damages
100 − 0	1 − .005	No damges

Net gain → L1

P(Net gain) → L2

Use 1-Var Stats with L1 & L2

E.V. = $\mu = \bar{x} =$ \$95

Airline makes \$95 per policy sold.

Jun 27-7:58 AM

A piggy bank has 3 Quarters and 7 Nickels.

Take 2 Coins, No replacement

NN NQ QN QQ

10¢ 30¢ 50¢

$$P(10¢) = \frac{{}_7C_2 \cdot {}_3C_0}{{}_{10}C_2} = \frac{21}{45} = \frac{7}{15}$$

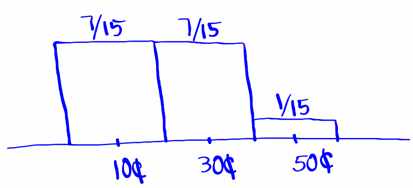
$$P(30¢) = \frac{{}_7C_1 \cdot {}_3C_1}{{}_{10}C_2} = \frac{21}{45} = \frac{7}{15}$$

$$P(50¢) = \frac{{}_7C_0 \cdot {}_3C_2}{{}_{10}C_2} = \frac{3}{45} = \frac{1}{15}$$

Total	P(Total)
10¢	7/15
30¢	7/15
50¢	1/15

Jun 27-8:05 AM

Total	P(Total)
10¢	7/15
30¢	7/15
50¢	1/15



Total → L1 , P(Total) → L2

Use [1-Var Stats] with L1 & L2 to find

$\mu = 22$ $\sigma = 12.220$ σ^2(exact) $\frac{448}{3}$

[VARS]
[5: Statistics]
[4: σ_x] [x^2] [Enter]
$149.\overline{3}$
[MATH] [1: ▶Frac] [Enter]

$\mu = 22$, $\sigma \approx 12$

68% Range → $\mu \pm \sigma$
→ [10 to 34]

Usual Range → $\mu \pm 2\sigma$
95% Range
→ [-2 to 46]

SG 14 & SG 15 ✓

Jun 27-8:12 AM

Binomial Prob. Dist: SG 16

1) n independent events

2) Each event has only two outcomes

$P(\text{Success}) = p$ $P(\text{Failure}) = q$

$p + q = 1$

p & q remain unchanged for all n events

3) x is # of Successes

$n - x$ is # of failures

$$P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}$$

$x = 0, 1, 2, 3, \ldots, n$

Jun 27-8:45 AM

You flip a coin 10 times. $n = 10$

$P(\text{Tails}) = .6$ $p = .6$

$q = .4 \rightarrow q = 1 - p$

P(exactly 5 Tails) $x = 5$

$P(x=5) = {}_{10}C_5 \cdot (.6)^5 \cdot (.4)^{10-5} = 252 \cdot (.6)^5 \cdot (.4)^5$

$P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}$ $= \boxed{.201}$

P(exactly 7 tails) $= P(x=7)$ $x = 7$

$= {}_{10}C_7 \cdot (.6)^7 \cdot (.4)^3$ ($n - x$)

$= 120 \cdot (.6)^7 \cdot (.4)^3 = \boxed{.215}$

Jun 27-8:50 AM

We randomly select 20 newborn babies.

$P(\text{Boy}) = .5$, $P(\text{Girl}) = .5$

$n = 20$
$p = .5$
$q = .5$

$P(\text{exactly } 12 \text{ boys}) = P(x = 12)$

$x = 12$

$n - x = 8$

$= {}_{20}C_{12} \cdot (.5)^{12} \cdot (.5)^{8}$

$= 125970 \cdot (.5)^{12} \cdot (.5)^{8}$

$= \boxed{.120}$

Using TI:

[2nd] [VARS] ↓↓ ... [binompdf(] 20, .5, 12 [Enter]
(n, p, x)

No Menu

With Menu
Trials: 20
P: .5
x-Value: 12
[Paste] [Enter]

Your work

$P(x=12) = \text{binompdf}(20, .5, 12)$

$= \boxed{.120}$

Jun 27-8:57 AM

You are taking a multiple-choice exam with 25 questions. $\boxed{n = 25}$

Each question has 5 choices, and only one correct choice. $p = \frac{1}{5} = .2$ $q = \frac{4}{5} = .8$

You are making random guesses.

$P(\text{guess } 8 \text{ correct answers}) = P(x = 8)$

$= \text{binompdf}(25, .2, 8)$

$= \boxed{.062}$

$P(\text{guess } 10 \text{ correct answers}) =$

$P(x = 10) = \text{binompdf}(25, .2, 10) = \boxed{.012}$

Jun 27-9:05 AM

P(guess at most 8 correct answers)

$x \le 8$

$= P(x \le 8) = \textbf{binomcdf}(25, .2, 8)$

$= \boxed{.953}$

P(guess at most 10 correct answers)

$x \le 10$

$= P(x \le 10) = \textbf{binomcdf}(25, .2, 10)$

$= \boxed{.994}$

Jun 27-9:10 AM

You flip a fair coin 100 times. Success is to land tails.

$n = 100$
$p = .5$
$q = .5$

1) P(land exactly 60 tails)

$= P(x = 60) = \textbf{binompdf}(100, .5, 60)$

$= \boxed{.011}$

2) P(land fewer than 60 tails)

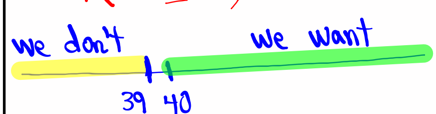
$= P(x < 60) = P(x \le 59) = \textbf{binomcdf}(100, .5, 59)$

$= \boxed{.972}$

3) P(land at least 40 tails) Total Prob.

$= P(x \ge 40) = 1 - P(x \le 39)$

$= 1 - \text{binomcdf}(100, .5, 39)$



$= \boxed{.982}$

Jun 27-9:15 AM

UPS says prob. that any item arrives on time is .85.

$p = .85$

$q = .15$

We randomly select 60 items. $n = 60$

Success is to arrive on time or sooner.

P(exactly 55 are on time) $= P(x = 55)$

$= \text{binompdf}(60, .85, 55)$

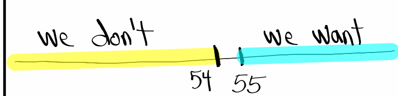
$= \boxed{.054}$

P(at most 55 arrive on time) $= P(x \le 55)$

$= \text{binomcdf}(60, .85, 55)$

$= \boxed{.958}$

P(at least 55 arrive on time) $= P(x \ge 55)$



$= 1 - P(x \le 54)$

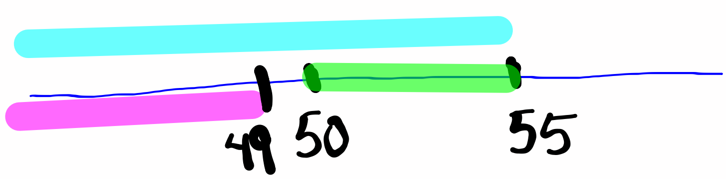
$= 1 - \text{binomcdf}(60, .85, 54)$

$= \boxed{.097}$

Jun 27-9:24 AM

P(between 50 & 55, inclusive, arrive on time)

$P(50 \le x \le 55) = P(x \le 55) - P(x \le 49)$



$= \text{binomcdf}(60, .85, 55) -$

$\text{binomcdf}(60, .85, 49)$

$= \boxed{.674}$

Jun 27-9:33 AM

Consider a binomial Prob. dist with $n=400$ and $p=.8$

1) $q = 1-p = 1-.8 = \boxed{.2}$

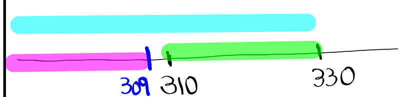
2) $np = 400(.8) = \boxed{320}$

3) $npq = 400(.8)(.2) = \boxed{64}$

4) $\sqrt{npq} = \sqrt{64} = \boxed{8}$

Let x be # of Successes,

5) $P(310 \le x \le 330) = P(x \le 330) - P(x \le 309)$



= binomcdf(400, .8, 330) − binomcdf(400, .8, 309)

$= \boxed{.811}$

Jun 27-9:38 AM

Class QZ 10

Use the chart below

x	$P(x)$
2	.15
4	.25
6	.35
8	.25

$\mu \approx 5$, $\sigma \approx 2$

Usual Range (95% Range)

$\mu \pm 2\sigma \Rightarrow$ 1 to 9

$x \rightarrow L1$, $P(x) \rightarrow L2$

[1-VAR Stats] with L1 & L2

Find

✓ 1) $\mu = 5.4$

✓ 2) $\sigma = 2.010$

Round to 3-decimal Places

3) σ^2(exact) $= 4.04$ or $\frac{101}{25}$

[VARS] [5: Statistics] [4: σ_x] [x^2] [Enter]

[Math] [1: ▸Frac] [Enter]

Jun 27-9:47 AM

Binomial Prob. dist.

- Mean → $\mu = np$
- Variance → $\sigma^2 = npq$
- Standard deviation → $\sigma = \sqrt{\sigma^2}$

Consider a binomial Prob. dist with $n = 100$ & $p = .5$

$q = 1 - p = 1 - .5 = \boxed{.5}$

$\sigma = \sqrt{\sigma^2} = \sqrt{25} = \boxed{5}$

$\mu = np = 100(.5) = \boxed{50}$

$\sigma^2 = npq = 100(.5)(.5) = \boxed{25}$

68% Range
$\mu \pm \sigma \rightarrow$ 45 to 55

95% Range
$\mu \pm 2\sigma \rightarrow$ 40 to 60

$x \rightarrow$ # of Successes,

$P(40 \le x \le 60) =$ binomcdf(100, .5, 60) − binomcdf(100, .5, 39)

Reduce by 1

$= \boxed{.965}$

Jun 27-10:22 AM

You are taking a True/false exam with 400 questions. You are making random guesses.

1) $n = 400$
2) $p = \frac{1}{2} = .5$
3) $q = \frac{1}{2} = .5$

4) $\mu = np = 400(.5) = \boxed{200}$

5) $\sigma^2 = npq = 400(.5)(.5) = \boxed{100}$

6) $\sigma = \sqrt{\sigma^2} = \sqrt{100} = \boxed{10}$

7) 68% Range → $\mu \pm \sigma$ → 190 to 210

8) 95% Range → $\mu \pm 2\sigma$ → 180 to 220 Usual Range

9) 99.7% Range → $\mu \pm 3\sigma$ → 170 to 230

10) P(guess between 170 and 230 correct ans, inclusive)

$= P(170 \le x \le 230) =$ binomcdf(400, .5, 230) − binomcdf(400, .5, 169)

Reduce by 1

$= \boxed{.998}$

99.8%

Jun 27-10:30 AM

Prob. of anyone in LA is a Laker fan is .75.

80 people were randomly selected.

1) $n=80$ 2) $p=.75$ 3) $q=.25$

4) $\mu=np$ $=60$

5) $\sigma^2=npq$ $=15$

6) $\sigma=\sqrt{\sigma^2}$ $=\sqrt{15}\approx 4$

7) Usual Range $\mu \pm 2\sigma \Rightarrow$ 52 to 68 (95% Range)

Let x be # of Lakers fan

8) $P(52 \le x \le 68)$ = binomcdf(80, .75, 68) − binomcdf(80, .75, 51) = .973

9) P(at least $\frac{3}{8}$ of them are lakers fan) ($\frac{3}{8}\cdot 80$)

= $P(x \ge 30)$ = 1 − binomcdf(80, .75, 29) ≈ 1

10) P(at most $\frac{5}{8}$ of them are lakers fan) ($\frac{5}{8}\cdot 80 = 50$)

= $P(x \le 50)$ = binomcdf(80, .75, 50) = .009

Jun 27-11:06 AM

class QZ 11

Use the chart below

x	$P(x)$
1	.1
3	.2
5	.3
7	.35
9	.05

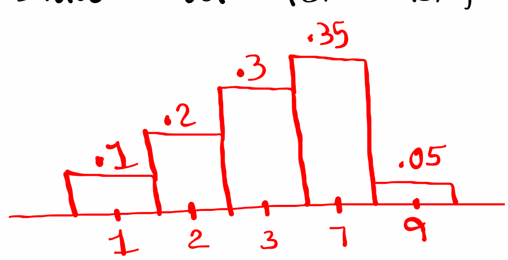
1) Find $P(x=9)$

$=1-[.1+.2+.3+.35]=.05$

2) Find $P(x>1)$

$=1-P(x=1)=1-.1=.9$

3) Draw Prob. dist. histogram



Jun 27-11:02 AM